

ELECTROWEAK SUDAKOV CORRECTIONS AT 2 LOOP LEVEL *

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Abstract

In processes at the energy much higher than electroweak scale, weak boson mass act as the infrared cutoff in weak boson loops and resulting Sudakov log corrections can be as large as 10%. Since electroweak theory is off-diagonally broken gauge theory, its IR structure is quite different from that of QCD. We briefly review recent developments on electroweak Sudakov and discuss on the exponentiation of Sudakov double logs and explicit 2 loop calculations in Feynman gauge.

1 Introduction

High energy experiments in TeV region are planned in the near future to obtain the hints to the fundamental problems in particle physics such as the mechanism of electroweak symmetry breaking, the gauge hierarchy problem and so on. The precision measurements in this region are expected to give us the important information to construct the scenario favorable up to Planck scale.[‡] In order to extract these information from experimental data, it is crucial to carry out the theoretical calculations at least in the same level of accuracy in the standard model and in other possible models. It is very important also for the estimation of the background of the production of the new particle.

Recently Ciafaloni and Comelli^{1,2} pointed out that in processes with energy much higher than electroweak scale, the “heavy particle masses” $M_W \simeq M_Z (\equiv M)$ acts as IR cutoff in weak boson loop integrals and resulting IR log corrections give large contributions to cross sections, comparable or larger than

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[‡]The possibility of the large extra dimension is not considered in this note.

QCD corrections. For example, 1 loop log corrections to $e^+e^- \rightarrow \mu^+\mu^-$ are³,

$$\sigma^\mu = \sigma^{tree} \times \left[1 + \frac{\alpha}{4\pi \sin^2 \theta_W} \left\{ \underset{\text{UV log}}{0.6 \ln \frac{Q^2}{\mu^2}} + \underset{\text{IR log}}{9.4 \ln \frac{Q^2}{M^2}} - \underset{\text{Sudakov log}}{1.4 \ln^2 \frac{Q^2}{M^2}} \right\} \right].$$

The second and the third terms which vanish in LEP energy region give the dominant corrections in TeV region.

Especially Sudakov double logarithms⁴ which come from the overlap of the soft and the collinear region of the loop integral give the dominant radiative corrections in the asymptotic regime (Their typical value at $\sqrt{S} \simeq 1\text{TeV}$ is $\frac{\alpha}{4\pi \sin^2 \theta} \log^2 \frac{S}{M_W^2} \simeq 6.6\%$). Since factors like $\alpha^n \log^{2n} \frac{S}{M_W^2}$ (LL), $\alpha^n \log^{2n-1} \frac{S}{M_W^2}$ (NLO), \dots , may spoil the perturbation expansion, it is necessary to control or resum these contributions to obtain a reliable and precise calculation in extremely high energy processes.

The infrared structure of gauge theory has been extensively investigated and it is well-known that Sudakov logs exponentiate for the form factor in QED^{4,5} and QCD⁶ and that they can be resummed in various physical processes⁷. The exponentiation of Sudakov logs in these cases results from the gauge symmetry and Lorentz symmetry in the factorization formula of cross sections⁸. In case of the electroweak theory, the situation is quite different. Since the gauge symmetry is broken and its breaking pattern is off-diagonal, the exponentiation of Sudakov logs in this case is a non-trivial matter. Several discussions on the all order behavior of Sudakov terms have been presented with different results^{9,10,11} and the explicit 2 loop calculations^{13,14,15} imply the exponentiation of Sudakov logs. From these discussions, we can see that the methods valid in QCD can not be applied straightforwardly to electroweak case and adequate modifications are needed.

The most striking feature of electroweak Sudakov is that they appear not only in the exclusive processes but also in the inclusive processes since non-abelian charge is not confined². In non-abelian gauge theory, it is known that Bloch-Nordsieck cancellations occur in leading power when both the sum of the final degenerate states and the initial color average are taken¹⁶. In QCD processes where initial color averages are always taken due to the color confinement, Sudakov double logs appear only in the end-point region of the phase space where the soft cancellation fails due to the suppression of the real emissions. On the other hand, initial state in electroweak processes are color (isospin) non-singlet and Sudakov logs remain even when the number of weak boson is not counted in the final state. Therefore, it can be said that the effects of symmetry breaking remain even we go up to the extremely high energy in which we usually consider that the gauge symmetry is restored.

In the following sections, we introduce the discussions on all order behavior of Sudakov logs and explain 2-loop calculation of the form factor in detail and discuss on other recent development.

2 Discussion on all order behavior of Sudakov logarisms

Discussion on all order behavior of Sudakov logarisms have been presented by several authors^{10,9,11}. The author of¹⁰ calculated all order $\ln^{2n} \frac{S}{M^2}$ terms in the self-energy in Axial gauge including only the weak boson loop and showed that Sudakov terms does not exponentiate due to the mixing effect¹⁰. Ciafaloni and Comelli⁹ estimated the leading log terms of the form factor in Feynman gauge using soft insertion formula which is the formalism developed in QCD¹⁷. Soft insertion formula means that we can obtain leading log terms by inserting the eikonal current at fermion legs, and estimating the diagrams with energy ordering. Their result was that the QED effects and electroweak effects factorize and Sudakov logs exponentiate in operator form but does not exponentiate numerically. Fadin *et.al.*¹¹ gave a general argument using the infrared evolution equation¹⁸, which is the differential equation with respect to the infrared cutoff derived from Gribov's Bremsstrahlung theorem. They derive two equations, one is for the region where $\mu \leq M_W$ with QED kernel and another is for $\mu \geq M_W$ with electroweak kernel. The exponentiation as the numerical value is concluded naturally from their recursive form.

3 Explicit calculation

Explicit 2 loop calculations of leading logarisms for the form factor are helpful to resolve the controversy discussed above and to construct the rigorous all order proof of exponentiation of electroweak Sudakov. We describe the similarity and the difference between this case and the QCD case concerning the cancellation of the non-exponential factors.

3.1 QCD case

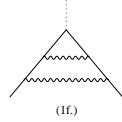
2 loop calculations of the leading singularities of the form factor were accomplished many years ago in QED²⁰ and QCD²¹.

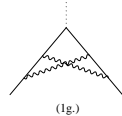
QCD double log corrections for the form factor come from vertex corrections in Feynman gauge. Using the mass regularization for infrared divergences 1-loop result is,

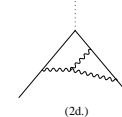


$$= -\frac{e^2 Q^2}{16\pi^2} \ln^2 \frac{S}{\lambda^2} ,$$

where C_F is the SU(3) Casimir operator for the fundamental representation and S is the momentum transfer. At 2 loop level, the leading double logs appear from the ladder diagram, the crossed ladder diagram, and the diagrams including triple gluon coupling as follows,



$$(If.) \quad = \frac{\alpha_s^2}{(4\pi)^4} \frac{1}{24} \ln^4 \frac{S}{\lambda} \times C_F^2 ,$$


$$(Ig.) \quad = \frac{\alpha_s^2}{(4\pi)^4} \frac{1}{12} \ln^4 \frac{S}{\lambda^2} \times (C_F^2 - \frac{1}{2} C_A C_F) ,$$


$$(2d.) \quad \times 2 \quad = -\frac{\alpha_s^2}{(4\pi)^4} \frac{1}{12} \ln^4 \frac{S}{\lambda^2} \times \frac{1}{2} C_A C_F .$$

Then the sum of these contributions are exactly a half of the 1-loop contribution squared. The non-exponential terms which appear in the crossed diagram and the diagram with the triple gluon coupling due to the non-abelian nature cancel each other, and the result is consistent with the exponentiation of the Sudakov logarithms.

3.2 Electroweak theory

2-loop explicit calculations have been done for self energy in axial gauge by Beenakker and Werthenbach and for the form factor of right-handed fermion in Feynman gauge by Melles¹⁴. Here we concentrate on the calculations of the form factor for left-handed fermions¹⁵.

The situation becomes more complicated in electroweak theory than QCD since the gauge symmetry is spontaneously broken and the pattern of the symmetry breaking is not diagonal. In physical processes photon must be treated in a semi-inclusive way since it is uncountable, on the other hand, weak bosons can be treated both inclusive and exclusive ways. The process we consider now

is the fermion pair production from the $SU(2) \otimes U(1)$ singlet source and the infrared divergences by photon loops are regularized by the fictitious photon mass λ . We must treat gauge bosons with different mass ("mass gap"). Within the leading log approximations, the fermion chirality is conserved and W and Z boson mass can be approximated to be equal and the field of unbroken phase *i.e.*, W and Z can be used though $S \gg M_W^M \equiv M$. The mixing effect comes from

$$[T^\pm, Q] \neq 0,$$

where T^\pm is the $SU(2)$ generator and $Q = T^3 + Y$ is the charge of fermion. For the right-handed fermion, the gauge group of electroweak interaction is reduced to $U(1)_Q \otimes U(1)_{Y-Q}$ and the exponentiation become a trivial matter. It also should be noted that if we take $M^2 = \lambda^2$, the mixing effects disappear and the calculation becomes same with that for unbroken $SU(2) \otimes U(1)$ gauge theory with regularization mass M and the exponentiation holds as QCD case.

The group factors for each SM boson exchange become,

$$\left\{ \begin{array}{l} \gamma \text{ exchange} : e^2 Q^2 \\ W \text{ exchange} : g^2 \sum_{a=1,2} T^a T^a \\ Z \text{ exchange} : \frac{g^2}{\cos^2 \theta_W} (T^3 - \sin^2 \theta_W Q)(T^3 - \sin^2 \theta_W Q) \\ \quad \quad \quad = g^2 T^3 T^3 + g'^2 Y^2 - e^2 Q^2 \end{array} \right.$$

Then, 1 loop result turn out to be,

$$\Gamma^{(1)} = 1 - \frac{1}{(4\pi)^2} (g^2 C_F + g'^2 Y^2 - e^2 Q^2) \ln^2 \frac{S}{M^2} - \frac{1}{(4\pi)^2} e^2 Q^2 \ln^2 \frac{S}{\lambda}.$$

Here C_F denotes the $SU(2)$ Casimir operator for the fundamental representation. At the 2 loop level, there appear 16 diagrams which are classified in 3 groups. That is, ladder diagrams, crossed ladder diagrams (Fig.1) and the diagrams including 3 gauge boson couplings (Fig.2). Here we use the following definitions for brevity,

$$\begin{aligned} e^2 Q^2 &\equiv \gamma, & (g^2 C_F + g'^2 Y^2 - e^2 Q^2) &\equiv (W + Z), \\ l &\equiv \frac{1}{2\sqrt{2}\pi} \ln \frac{S}{\lambda^2}, & L &\equiv \frac{1}{2\sqrt{2}\pi} \ln \frac{S}{M^2}, \end{aligned}$$

for the group factors and the loop factors respectively. After the several cancellations among different diagrams, the 2 loop results for ladder diagrams,

crossed ladder diagrams, and the diagrams including 3 point coupling turn out to be,

$$\begin{aligned}
\text{ladder} &= (1a.) + (1c.) + (1d.) + (1f.) \\
&= \gamma^2 \frac{1}{24} l^4 + 2\gamma(W+Z) \frac{1}{8} l^2 L^2 + (W+Z)^2 \frac{1}{24} L^4 \\
&\quad + \gamma(W+Z) \left[\frac{1}{6} L^4 - \frac{1}{3} L^3 l \right], \\
\text{crossed} &= (1b.) + (1e.) \times 2 + (1g.) \\
&= \gamma^2 \frac{1}{12} l^4 + \{(W+Z)^2 - g^4 \frac{1}{2} C_V C_F\} \frac{1}{12} L^4 \\
&\quad + \gamma(W+Z) \left[-\frac{1}{6} L^4 + \frac{1}{3} L^3 l \right] + 2g^2 e^2 Q T^3 \left[\frac{1}{6} L^4 - \frac{1}{3} L^3 l \right], \\
\text{3 point} &= [(2a.) + (2b.) + (2c.) + (2d.)] \\
&= g^4 \frac{1}{2} C_V C_F \frac{1}{12} L^4 + 2g^2 e^2 Q T^3 \left[-\frac{1}{6} L^4 + \frac{1}{3} L^3 l \right].
\end{aligned}$$

Here the underlined terms are the extra terms compared to QCD case which come from the mixing effect. After these terms cancels each other, the sum of these contributions becomes,

$$\begin{aligned}
\Gamma^{(2)} &= 1 - \frac{1}{16\pi^2} (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) \ln^2 \frac{S}{M^2} - \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{S}{\lambda^2}, \\
&\quad + \frac{1}{2!} \left[\frac{1}{16\pi^2} (g^2 C_2(R) + g'^2 Y^2 - e^2 Q^2) \ln^2 \frac{S}{M^2} + \frac{1}{16\pi^2} e^2 Q^2 \ln^2 \frac{S}{\lambda^2} \right]^2
\end{aligned}$$

which imply the exponentiation of Sudakov form factor.

We can see from this result that

- (1) γ must be included to extract the correct coefficient of $\ln^4 \frac{S}{M^4}$.
- (2) Non-exponential factor appear also in ladder diagrams.

The first point comes from the fact that the singularity from the photon which propagates inside the W and/or Z loop in Figs.1d and 2c is regulated by the W and/or Z mass¹⁴. The gauge invariant set including photon contributions must be included to extract the correct $\ln^2 \frac{S}{M^2}$ terms. The second is the important difference from QCD. In QCD case, the contributions of the ladder diagrams and similar part of the crossed ladder diagrams remain in final results and those of “3-point” diagrams act only as the counter term against the non-exponential term, while the sum of former two contributions can be extracted

from the ordered ladder diagrams. Therefore, the approach by soft insetion formula which is valid for QCD can not be applied straightforwardly to this case and modifications seems to be necessary..

4 Discussions

Electroweak log corrections become very important in the processes in TeV region. Since they give large corrections to the cross sections, It is crucial to control them in perturbation series to reduce the uncertainty of theoretical predictions in precision measurements and in the backgroud estimations for the signals of the new physics.

Exponentiation of Sudakov double logarisms seems to be valid from the discussion using the infrared evolution equation and the explicit 2-loop calculations. The resummation of next to leading logarisms were accomplished by Kühn, Penin and Smirnov¹² for fermion external line and by Melles for general external line and for the top Yukawa enhanced part^{19,22}.

As for the phenomenology, several works have been done. 1-loop[?], and 2-loop¹² Sudakov-type log corrections to $e + e- \rightarrow f \bar{f}$ and 1-loop effects in MSSM²⁴ were calculated. Denner and Pozzorini²⁵ presented the complete 1-loop electroweak log corrections for the general processes. Baur²⁶ discussed on the impact of electroweak Sudakov on the W boson mass measurement at LHC.

Above calculations indicate that NLO Sudakov effects are comparable with LO Sudakov effects at the energy around 1 TeV. Therefore, it can be said that we can not yet control the electroweak log corrections within the 1% level and much have to be done from now.

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1. P. Ciafaloni and D. Comelli, *Phys. Lett.* **B446** (1999) 278.
2. M. Ciafaloni, P. Ciafaloni and D. Comelli, *Phys. Rev. Lett.* **84** (2000) 4810; *Nucl. Phys.* **B589** (2000) 359; and hep-ph/0007096.

3. M. Beccaria, P.Ciafaloni, D. Comelli, F. Renard and C. Verzegnassi, *Phys. Rev.* **D61** (2000) 073005.
4. V. V. Sudakov, *Sov. Phys. JETP* **3** (1956) 65.
5. D. Yennie, S. Frautschi and H. Suura, *Ann. Phys.***13** (1961) 379.
6. See *e.g.* J. C. Collins, in *Perturbative Quantum Chromodynamics* ed. A. H. Mueller (World Scientific, Singapore,1989) P. 573; and references therein.
7. See *e.g.* G. Sterman, in *QCD and Beyond* ed. D. Soper (World Scientific, Singapore, 1996)P. 327; and references therein ; hep-ph/9606312.
8. H. Contopanagos, E. Laenen and G.Sterman, *Nucl. Phys.* **B484** (1997) 303.
9. P. Ciafaloni and D. Comelli, *Phys. Lett.* **B476** (2000) 49.
10. J. H. Kühn and A. A. Penin, hep-ph/9906545.
11. V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles, *Phys. Rev.* **D61** (2000) 094002.
12. J. H. Kühn, A. A. Penin and V. A. Smirnov, *Eur. Phys. J.* C17 (2000) 97.
13. W. Beenakker and A. Werthenbach, *Phys. Lett.***B489** (2000) 148.
14. M. Melles, *Phys. Lett.***B495** (2000) 81.
15. M. Hori, H. Kawamura and J. Kodaira, *Phys. Lett.***B491** (2000) 275.
16. See *e.g.* M. Ciafaloni, in *Perturbative Quantum Chromodynamics* ed. A. H. Mueller (World Scientific, Singapore,1989) P. 491.
17. A. Bassetto, M. Chiafaloni and G. Marchesini, *Phys. Rept* **100** (1983) 201 ;
S. Catani and G.Marchesini, *Nucl. Phys.* **B249** (1985) 301.
18. V. N. Gribov, *Yad. Fiz.* **5** (1967) 399 [*Sov. J. Nucl. Phys.* **5** (1967) 280];
R. Kirschner and L. N. Lipatov, *JETP* **56** (1982) 266; *Phys. Rev.* **D26** (1982) 1202 ;
L. N. Lipatov, *Nucl. Phys.* **B307** (1988) 705;
V. Del. Duca, *Nucl. Phys.* **B345** (1990) 369.
19. M. Melles, *Phys. Rev.* **D63** (2001) 034003.
20. R. Jakiew, *Ann. Phys.* **48** 292;
P. M. Fishbane and J. D. Sullivan, *Phys. Rev.* **D4** (1971) 458;

- T. Appelquist and J. R. Primack *Phys.Rev.* **D4** (1971) 2454 etc.
21. J. Carrazone, E. Poggio and H. Quinn, *Phys. Rev.* D11 2286.
 J. M. Cornwall and G. Tiktopoulos, *Phys. Rev.* **D13** (1976) 3370;
 J. Frenkel, M. -L. Frenkel and J. C. Taylor, *Nucl. Phys.* **B124** (1977)
 268; and references therein.
 22. M. Melles, hep-ph/0012157.
 23. M. Beccaria, P.Ciafaloni, D. Comelli, F. Renard and C. Verzegnassi,
Phys. Rev. **D61** (2000) 011301.
 24. M. Beccaria, F. Renard and C. Verzegnassi, hep-ph/0007224.
 25. A. Denner and S. Pozzorini, hep-ph/0010201.
 26. U. Baur, hep-ph/0007287.

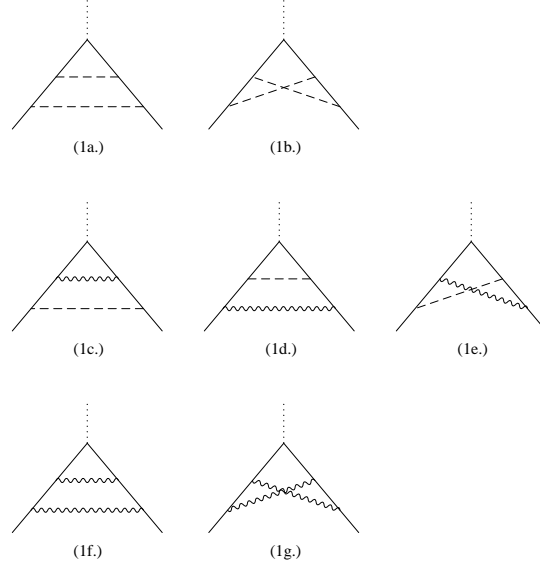


Figure 1: The ladder and crossed ladder diagrams. The dashed (wavy) line represents the photon (W and/or Z) with the mass λ (M).

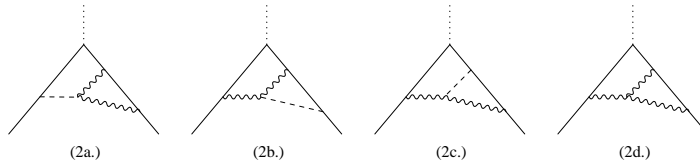


Figure 2: The diagrams which have the triple point couplings. The meaning of lines is the same as in Fig.1.